GAMES AS FACILITATING TOOLS IN TEACHING OF FUNCTIONS IN A PUBLIC SCHOOL IN MANAUS

Adalberto da Silva Cavalcante

SUMMARY

The teaching and learning of Mathematics has proven to be a major challenge, especially with regard to the content of functions in High School, the understanding of which is of great relevance, as it facilitates the learning of other subjects and the application in everyday situations. The present work aims to show the importance of using games in the Mathematics discipline, as facilitators of understanding and better assimilation of the concepts necessary for the proper learning of the content of functions, as well as stimulating teamwork and compliance with rules. . With the aim of minimizing learning difficulties in a 1st year high school class, with grade/age distortion and coming from a project called "Avançar", at the Alice Salerno Gomes de Lima State School, located in the city of Manaus, State of Amazonas, we sought a motivating strategy for teaching functions based on the theories of David Tall and Shlomo Vinner (1981), who in their studies used the idea of the function machine to introduce the concept of function and thus create an image conceptual. We adapted and applied nine games, namely: Adapted Checkers, Figures Function Machine, Find the Exit Function Machine, Determine the Function Function Machine, Naval Battle with Cartesian Coordinates, Function Concept Trail, Functional Stop, Amazonas Gives Luck Functions and Winding Path. These games aim to create a conceptual image consistent with the cognitive structure that is associated with the concept, including all mental images and properties associated with them as well as the processes involved. Games are a consistent didactic instrument, because according to theorists Borin, Grando, Nogueira and Lara, among others, pedagogical work with games involves deductive reasoning for the play, for argumentation and exchange of information, in addition to allowing proof of efficiency of thought-out strategies. For this research, we used the categories of games proposed by Lara (2003): Construction games, Training games, Deepening games and Strategic games. Before the presentation of the games, a test on the content of functions was administered. The games were presented to the students in ten weekly meetings. At the end of each meeting, observations were made on the students' performance. Such observations allowed us to realize that even though students had some difficulties related to mathematical operations with integers and rational numbers, the result obtained was very satisfactory. The analysis of the pre-test and post-test showed that the students began to have a more consistent conceptual image about functions, progress in graph analysis, identification of important elements of the function and satisfactory interpretation of everyday problems that involved the concept of function .We hope that this work has contributed to improving the teaching and learning of Mathematics in the classroom.

Key words: Learning; Concept; Teaching; Functions; Games; Mathematics.

1. INTRODUCTION

Mathematics is a science that participates in our lives at all stages of development. Everything we do, or plan during our lives, involves aspects inherent to this discipline, among which we can consider numbers, finances, geometric figures, distances, measurements, among other factors that guide this discipline.

Therefore, we can consider Mathematics as a subject of great relevance in the training and insertion of students into the job market. However, we found that in High School the dropout and failure rates in this subject grow every year, and the majority of students who continue studying show a lack of interest in learning Mathematics. Furthermore, as a public school teacher for over twenty years, I have observed that students arrive at High School without having mastered the minimum prerequisites necessary to understand Mathematics in High School, such as mastering the four operations numerical skills or the skills needed to solve problems. However, the main fact observed is the lack of students' skills in writing, reading and interpreting texts, which are fundamental tools for all subjects, and in the case of Mathematics, especially for the content of functions. The concept of function offers a set of resources that will facilitate the learning of other mathematical content.

At Escola Estadual Prof.ª. Alice Salerno Gomes de Lima[1], where I work in the afternoon, as a Mathematics teacher, the vast majority of students attend high school, as an access to the job market or admission to a university. One of the probable reasons for the difficulties detected may be the inadequate form of teaching in Elementary School, as the students come from another school that uses the process of advancing their studies to correct the age/grade distortion. So what to do? And how to change this situation? With the aim of changing this situation, we seek a motivating teaching strategy for the study of Mathematics. And for this purpose, we present students with a series of games that contain activities and problem situations involving the teaching of functions. The purpose of carrying out this research was to investigate the use of games as a didactic strategy and its consequences on the learning of 1st year high school students, concerning the main elements of 1st and 2nd degree polynomial functions. The games were applied in

1 School belonging to the public network of the State of Amazonas. It offers secondary education to 860 students. Located at Rua 7, nº 12, Conj. Castelo Branco, Neighborhood: Parque 10, in Manaus-Amazonas.

a class of 36 students, from the 1st year of high school, from Escola Estadual Prof.[a]. Alice Salerno Gomes de Lima, in the city of Manaus, State of Amazonas and the results indicated that the use of these games as a teaching strategy served as a stimulus for students and to advance activities in the classroom, thus contributing to a better understanding and fixing the content of functions.

Games will be presented that address the concept of function, polynomial function of the first degree and polynomial function of the second degree. The creation of these games is motivated by our concern as an educator, the desire to act in a creative and participatory way in the classroom, in relation to these contents and the disagreement with the pedagogical practice that has been used, in which the teacher appears as the central figure. keeping the student in a passive posture, just reproducing what was taught. The purpose is to create a beneficial and motivating environment in the classroom, allowing the student to participate in the process of constructing the concept of function and exploring its properties. To develop the games, considerations were initially made on the questions explained by Flemming and Collaço de Mello (2003) for teachers who want to use games in their classes: “(1) What is the objective I intend to achieve? Do I know a suitable game? 2) Will I need to make an adaptation? 3) What materials are needed to apply the chosen game? How to apply it?”

The use of games is a technique that can facilitate learning, as it makes teaching more interesting in order to enable students to build autonomy, motivating them to build more consistent and contextualized mathematics knowledge, as Smole states:

> Working with games in mathematics classes, when well planned and guided, helps the development of skills such as observation, analysis, raising hypotheses, searching for assumptions, reflection, decision-making, argumentation and organization, which are closely related to so called logical reasoning. (SMOLE*et al.*, 2008, p. 9).

Therefore, it is necessary to recover the meaning of what has been taught for a long time. According to Moreira (1999), we know the importance and need to make students enjoy acquiring mathematical knowledge in a meaningful way. For a long time, teaching and learning Mathematics has been a great challenge for teachers and students. According to Demo (2002), this subject is considered a “bogeyman”, as the teacher is just a transmitter of information and the students only copy, memorize and pass it on for assessments, and with this they have a view that mathematics it is very difficult to to understand. Therefore, the teacher cannot just think that students, with each passing year, arrive without the basic knowledge necessary to continue their studies. It first needs to generate means for students to have learning that makes sense, so that they feel interested and motivated to become authors of their knowledge, in order to have a satisfactory academic performance, with the production of material for use, or new mechanism that generates student support. The teacher of this subject has to play the role of helping students and making them enjoy Mathematics, improving their self-esteem, seeking new work strategies and making students see the importance of the subject in their lives. For this cognitive growth to occur, it is necessary to have an environment of respect and trust in which activities become pleasurable. For Lara (2003):

> Mathematics will only lose its aura as a bogeyman discipline when “we educators focus all our efforts so that teaching Mathematics is: developing logical reasoning and not just copying or exhausting repetition of standard exercises; stimulate independent thinking and not just mnemonic ability; develop creativity and not just transmit ready-made knowledge; develop the ability to manage real situations and solve different types of problems and not continue in that “sameness” that we experienced when we were students. (LARA, 2003, p. 18- 19).

2 THEORETICAL FOUNDATION

To change the erroneous view of Mathematics, the creation and use of games can be used as a pedagogical practice, providing more interesting and motivating learning for the student. According to Alves, (2001, p. 25), “[...] the game can establish concepts, motivate students, foster solidarity among colleagues, develop critical and creative sense, stimulate reasoning, discover new concepts”. By using games as a teaching strategy, satisfactory results can be achieved for students, as they will need to organize themselves, as necessary, defend positions and respect the opinions of other students during the implementation of the activity, all in a relaxed way. , as Borin (1996) mentions, in his work on the relevance of group games:

> As a work strategy, we chose group games for their playful aspect that can motivate and arouse student interest, making learning more attractive. Based on mistakes and successes and the need to analyze the efficiency of each strategy, constructed to achieve victory in the game, the development of reflective reasoning in those who play is encouraged. The same author says that it is necessary to explain to students why we are working with games. Our concern was to explain to students the objective why games were being adopted as a strategy in the classroom. We also made it clear that the intention was to use the games as if they were problems to be solved and, as they were

> By playing, they would realize the Mathematics that was present in this process. (BORIN, 1996, p. 3).

Therefore, the teacher needs to reflect on his practice, so that he can calmly apply this methodology and the student will also need some necessary, relevant observations, to have a minimum basis on the content that will be applied, so that he has a fixation better content and interaction with companions during the game.

Although it is not widely used in high school, the practice of games has a positive aspect, which is the replacement of the traditional teaching model in the classroom, in which the student alone needs to transform content into knowledge using exercises given by the teacher. This methodology gives students the fear they have in relation to Mathematics and the feeling of inability to solve mathematical questions. In this sense, BORIN (1996) highlights the advantages of using games:

> Another reason for introducing games in Mathematics classes is the possibility of reducing the blockages presented by many of our students who fear mathematics and feel unable to learn it. Within the game situation, where a passive attitude is impossible and motivation is high, we noticed that, at the same time that these students speak mathematics, they also show better performance and more positive attitudes towards their learning processes. (BORIN, 1996, p. 9).

This proposition is also in accordance with National Curricular Parameters (BRASIL, 1997, p. 36), which serve as a theoretical support for this teaching and learning proposal, "[...] participation in group games also represents a cognitive achievement , emotional and social for the child, and a stimulus for the development of their logical reasoning", helping in a creative and attractive way to develop skills and abilities for the full development of students.

2.1 GAME AS A TEACHING RESOURCE

The mathematical game used as a teaching resource is capable of promoting more interesting teaching and more dynamic learning, making classes more attractive and providing challenges, showing that Mathematics can become interesting and facilitate the understanding of mathematical content in this sense. Grando (2000) highlights:

> When analyzing the attributes and/or characteristics of the game that could justify its inclusion in teaching situations, it becomes clear that it represents a playful activity, which involves the player's desire and interest in the game's own action, and moreover, it involves the competition and challenge that motivate the player to know their limits and their possibilities of overcoming such limits, in the search for victory, acquiring confidence and courage to take risks. (GRAND, 2000 p. 24).

From this perspective, several researchers in the area of Mathematics Education have studied the potential of the game in the Mathematics teaching-learning process and highlight the importance of this methodological resource in the classroom. It is worth highlighting that the act of playing involves the affective aspect, which is the most important element in involvement in the game. This statement is in agreement with Oliveira (2007) when stating that:

> Teaching Mathematics is developing logical reasoning, stimulating independent thinking, creativity and the ability to solve problems. “We, as mathematics educators, must look for alternatives to increase motivation for learning, develop self-confidence, organization, concentration, stimulating socialization and increasing the individual's interactions with other people (OLIVEIRA, 2007, p. 5).

Therefore, the use of games and curiosities in teaching Mathematics aims to make students show interest in learning this subject, changing the classroom routine and promoting the interest of the student involved. Learning through the use of practical activities in our daily lives, which are actually useful for the student and are also interesting, not because they need to do well in the subject, but because they enjoy it. By analyzing the possibilities of games in teaching Mathematics, we noticed several moments in which students carry out activities with games in their daily lives outside of the classroom. Many of these cultural and spontaneous games use mathematical notions that are simply experienced during the game.

Teaching Mathematics is developing logical reasoning, stimulating independent thinking, creativity and the ability to solve problems. In this sense, Nogueira (2005) advises that:

> [...] pedagogical work with games involves deductive reasoning for the play, for argumentation and exchange of information, in addition to allowing proof of the efficiency of planned strategies. They rescue the fun of the classroom and contribute to reducing the blockages presented by children and adolescents who fear Mathematics and feel unable to learn it, as they now have experience that learning is an interesting and challenging activity (NOGUEIRA,2005, p. 53).

Therefore, by using games and curiosities in teaching Mathematics, we can make students enjoy learning this subject, change the classroom routine and arouse the interest of the student involved because according to Golbert (2002):

> It is up to the teacher to help students acquire the cultural tools that enable them to reflect on their own intuitions and experiences and communicate them, articulating their ideas, building richer understandings. This means that it is the teacher's responsibility to find ways to bridge the gap between the students' usual language and the more abstract mathematical conventions (GOLBERT, 2002, p. 8).

For Kishimoto (1997), constructivist theories were the first that teachers relied on to introduce games into teaching environments, in order to make students discover certain concepts. Almeida (1984), in his speech regarding the relevance of the game for education and its didactic benefits, guarantees that such resources have become essential factors in promoting meaningful learning, as several concepts, considered difficult, when applied through of games proved to be easier to understand. Lara (2003) highlights that the game is a powerful activity within the scope of participatory learning, in order to achieve the proposed objectives.

The game not only has the capacity to make classes more dynamic, but it also has the capacity to be an instrument for teachers to become capable of identifying their students' main difficulties and carrying out more effective learning diagnoses. The elaboration of mathematical knowledge through games within the school brings several advantages, because when playing, the student does so in a pleasurable way and makes a spontaneous and voluntary effort to achieve the objective (result). According to PCNs (1998), a relevant advantage in games is the challenge, which makes students feel more interest and pleasure in the subject. Therefore, games are essential for society to have individuals with the ability to seek solutions, face challenges, create strategies and become critical people. The use of games as a methodology for teaching and learning in the classroom has been happening slowly, as students need time to get used to the new methodologies. It is necessary for the teacher to be a mediator of the construction of learning when using them, as it must be provided in an environment where students can create, dare, challenge and prove. According to Grando (2000):

> The Mathematics teacher presents himself as one of those largely responsible for the activities to be developed in the classroom. Therefore, any necessary change to be made in the mathematics teaching-learning process will always be linked to the teacher's transformative action (GRAND, 2000, p.28).

In this way, it is clear that the interest in adapting new pedagogical methods, aimed at student learning, must come from the teacher, followed by the school and the students. Sometimes, it will be necessary for the teacher to intervene in the elaboration of the contextualization so that the class makes sense, becomes more attractive, gaining more specific knowledge and attracting the student's interest, and brings already existing knowledge, in order to increase more interest in the content and a motivation for learning in a playful way, which will enable the exchange of ideas with the class and the teacher, becoming the subject of the mathematics learning process.

3 CONCEPTUAL IMAGE AND CONCEPT

Theoretical bibliographical research was carried out with the aim of improving the theoretical foundations related to the teaching and learning process of mathematical thinking, in particular, the concept of functions. To help with these questions, the theoretical principles present in the notions of concept definition and concept image will be used, we will use the propositions made by researchers David Tall and Shlomo Vinner (1981). The two theorists defend the idea that a given concept could not be worked on based on its usual definition, therefore, for a given concept to make sense and be truly understood, the student must be familiarized with it prior to formalization. For Tall and Vinner (1981), the emergence of the Image Concept:

> [...] describes the entire cognitive structure that is associated with the concept, includes all mental images and properties associated with them and the processes. It is developed over the years through experiences of all types, changing both when the individual encounters new stimuli and when they mature (TALL; VINNER, 1981, p.152).

It is believed that when the student is encouraged to think about a certain object, several mental representations arise, such as images of visual representations, impressions, experiences and properties, which can be elaborated by the students through thought processes about mental representations. , which according to Tall and Vinner (1981) are called image concept. In this way, we will analyze the contributions of a teaching proposal based on the concepts of images and concept of definition for the study of functions in high school.

For Vinner (1983), the formation of concepts is of great importance for the psychology of learning. When working with this issue, one has to be aware of some difficulties: the first in relation to the notion of the concept itself and the second is to verify when a concept is correctly formalized in the mind of an individual. To explain this cognitive process, the notions of concept image and concept definition developed by Tall and Vinner (1981) will be based on this. The main idea consists of the following: when the student hears the name of a concept, he generates a stimulus in order to trigger his memory, described as an image concept. In this way, it can be stated that the concept of image is something that is evident in our mind in order to associate a non-verbal thing, which, according to Tall and Vinner (1981), non-verbal refers to visual representations, impressions or experiences referring to the name of the concept. Note that these representations do not verbal forms can be translated into verbal forms, however, the latter are not always accurate or the first to be remembered by our mind. For example, when we say in a function that x takes the value of the number a, the image may come to an individual's mind **( ) =** **The.**

The term concept image shows the total cognitive structure that is related to the concept. This total structure will represent the set of all images, properties or processes that the individual has ever had contact with, which will be associated with the concept. Therefore, it is clear that as the individual obtains new experiences over time related to a concept, the images are enriched, “expanding” the image concept. A definition concept is understood as an oral definition that precisely explains a concept. Therefore, we can state that the concept definition represents the formal mathematical definition. Tall and Vinner (1981) emphasize that for an individual to acquire a concept it is necessary to form a concept image of it. Only the definition concept (the effective definition) does not give us a guarantee of the true understanding of the concept. We remember that for some concepts, certain individuals, can have at the same time a definition concept and an image concept. Furthermore, some concepts can even be introduced with their definitions which, in this situation, help in the formation of the image concept.

Consequently,the definition serves as support in the construction of the image concept. However, often, even with support for the construction of the definition, what happens is that individuals, after forming the concept image of a concept, no longer use the concept definition. Therefore, after the formation of the image concept, the definition may remain inactive or even be forgotten. Vinner (1983) created an enlightening model for the construction of mathematical knowledge that is supported by the relationships that exist between the image concept and the definition concept. It guarantees that each of the concepts has distinct cells in the cognitive structure, one for the definition concept and the other for the image concept. If no meaning is associated with a concept, the image concept cell is empty. This happens in several situations where the concept is just memorized without meaning to the individual. This could be considered a decisive point with regard to the process of teaching and learning the concept of function, as exemplified by Iezzi and Murakami (2004):

> Given two non-empty sets A and B contained in R, a relation ƒ from A to B is called an application of A to B or a function defined in A with images in B if, and only if, for all x ∈ A exists a single y ∈ B such that (x, y) ∈ ƒ. (IEZZI; MURAKAMI, 2004, p. 81).

Definitions like this are usually introduced early, either by teachers or teaching materials in high school. Even though it is more common to recognize the idea that, to generate the improvement of an image concept, one must carry out various situations so that students can accurately capture the definition concept, what we see on a daily basis are attempts which do not show the intended effectiveness. It is at this point that students' greatest difficulties usually appear. In several situations, when students manage to accurately memorize the definition, without identifying its meaning, they end up constructing (when possible), based on this definition concept, a vague image concept normally associated with the expression "**x Є A**" or the notation used to represent a function. It is worth mentioning that the acquisition of a finite definitional concept applicable only to a number of particular examples may also occur.

The model designed by Vinner (1983) suggests the existence of a relationship between the definition concept and the image concept, even though both can be constructed independently. There is the possibility that a given individual constructs, from several examples, an image concept that may change as this individual comes into contact with situations that do not appear in the initially constructed examples. But it can also happen that even in the face of new situations that require changes and/or adaptation of the image concept, it remains unchanged, that is, the teacher's definition remains temporarily in the cell and can be forgotten or distorted over time.

The same way can occur when a concept is placed through its definition. The image concept cell, which is initially empty, is filled with examples and explanations that are being given. However, it is not capable of expressing all aspects of the definition concept. It is noted that the excess in definitions and demonstrations that are presented in a rigorous and mechanical way does not contribute, in the vast majority of cases, in a positive way to the acquisition of the concept of definition and the concept of image. The demonstration must be constructed through significant processes capable of presenting images that help the acquisition of the image concept and give meaning to the definition concept or vice versa. For Tall and Vinner (1981), care must be taken with the way in which the model is introduced into some mathematical activities, as researchers recognize that the formation of concepts does not always occur in the same way because it is linked to the subjects' performance. involved. Due to this, they suggest applying activities aimed at the construction and identification of concepts in order to put the student in contact with cognitive activities that allow the activation of two cells: the image concept cell and the definition concept cell.

Tall (1992) states that when students are initially confronted with mathematical concepts, it is infallible that they will find exclusively a limited range of possibilities that emphasize their conceptual images, and for the construction of an appropriate cognitive foundation we must start from concepts already known by the students. students that serve as support for the growth of mathematical knowledge, as stated by Tall (1992):

> Instead of initially dealing with formal definitions that contain elements unfamiliar to the student, it is preferable to try to find an approach that builds concepts that have the dual role of being familiar to students and also providing the basis for further mathematical development. I call such concepts cognitive roots. These are not easy to find – they require a combination of extensive research (to find what is appropriate for the student at the current stage of development) and mathematical knowledge (to be sure of the long-term relevance of mathematical terms (TALL, 1992, p. 04).

Learning the concept of function in many cases occurs through examples that use formulas. Tall and Vinner (1981) claim that in such a case the conceptual image may have a restricted notion, exclusively involving formulas, in which the conceptual definition remains mostly inactive in the cognitive structure. At first, the student in this position can fully function with his restricted notion, which is suited to his restricted context. However, later, when functions appear to be defined in broader contexts, you may be unable to deal with them. However, the person responsible for this situation is the teaching program itself.

We believe it is important for school institutions to train critical citizens to act within society. Therefore, it is necessary for students to develop their reasoning and try to use their learning to solve daily problems through a quality education in which the elaboration of knowledge takes place in an integrated, varied way and encourages the growth of images and definitions. students' concepts.

4 FUNCTION

Several concepts presented during our school life can be related to everyday situations. The idea of function is related to daily situations found in magazines, newspapers and television news, in addition to being present in various situations in society.

human activity, such as the price paid at the gas station to fill up with fuel. This knowledge can be used in classes to teach the concept of Function, generating interest and motivation in the student to obtain new knowledge.

According to the High School PCNs (BRASIL, 1999), part of Mathematics teaching is to ensure that the student gains a certain flexibility to deal with the concept of function in different situations. Through a variety of problem situations, the student can be encouraged to seek a solution, adapting their knowledge about functions and thus building a model for interpretation and investigation in Mathematics.

Functions, which are special relationships, are present throughout the school curriculum, being found in arithmetic, algebra, geometry and probability. The concept of function is of great importance, as it mathematically relates various situations found in the real world. Given this aspect, Akkoç and Tall (2005) reiterate that curriculum designers assume that students can conceptualize the definition of function after studying various representations.

According to Iezzi and Murakami (2004), in Mathematics if 'x' and 'y' are two variables such that for each value attributed to 'x' there will be a single corresponding value for 'y', in this case, it is said that 'y' is a function of 'x'. The set of values that can be assigned to 'x' is called the domain of the function. For the variable 'x', give the name independent variable. The value of 'y', obtained from the value assigned to 'x', we call the image of 'x' by the function and is denoted by f(x). The variable 'y' is called the dependent variable, because its values depend on the values that 'x' takes. The set called the image of the function is made up of the values that 'y' takes, in correspondence with the values of 'x'. According to Akkoç and Tall (2002):

> For a mathematician, the notion of function is a model of simplicity. What could be simpler than the idea that 'we have two sets and each element in the first is linked to precisely one element in the second'? Not only is the definition mathematically simple, for the mathematician it provides access to an enormous complexity of ideas. Some students are able to construct this delicate combination of simplicity and complexity. For others, however, the situation is quite different. How do they respond when the notion of function is introduced? They bring their implicit understanding of the language and all their previous experiences to bear on the task. The result for them is a highly complicated series of personal meanings that at the same time helps and hinders their interpretations of the mathematical concept (AKKOÇ; TALL, 2002, p. 01).

One of the difficulties encountered by students when learning Mathematics according to Akkoç and Tall (2005) is that students' cognitive development does not reach the logic of mathematics. Only a small proportion of students stick to learning the concept of function, while the majority only work with the concept in a disconnected way, which reveals an irregular combination between the curriculum and the students' cognitive structures.

For a long time, the complexity of the concept of function caught the attention of researchers. Because the difference between the conceptual definition used by mathematicians and the conceptual image generated in the students' minds was proven by Vinner (1983) apud Akkoç and Tall (2002), revealing that the majority of students use their own definitions for the concept of function, giving generally wrong meanings to the term.

According to Tall (1992), initially instead of working on formal definitions with elements practically unknown to students, the best would be to look for an approach that builds concepts that are familiar to them and serve as a basis for future mathematical development.

According to Tall (2000-a), the concept of function can be inserted in a way that is theoretically more efficient, thus seeking to develop the general foundations that are associated with other theories of cognitive development in mathematics teaching.

When we emphasize the many representations of the concept of function such as: mathematical sentence, formula, graph, relationship between variables, among other things, the main idea of function as a process is often neglected. For example, in the situation, although graphs are often represented as a great way to study and understand a function, few students relate the graph to the functional process that is implied, seeing the graph only as an inert object.

Students create models for the concept of function in the same way they create models for everyday concepts, and although the concept of function runs through several branches of mathematics and has a prominent position in its development, its teaching is complex. at school. From this perspective, Bakar and Tall (1991) indicate that “Modern Mathematics” introduced the concept of function at school only in terms of the domain of scale and the relationship between the elements, but several students demonstrated difficulties in understanding this notion. Even though we can show our students general concepts such as the domain in which the function is defined and the range of possible values, such terms do not seem to be understood.

According to Tall (1988), “Modern Mathematics” understood that if students elaborated definitions and mathematical deductions correctly, they would learn mathematics. However, even though all this was done, difficulties remained. Careful study suggests that such difficulties do not necessarily correspond to a lack of knowledge on the part of students, but to a natural human phenomenon that is evident in all of us.

Bakar and Tall (1991) carried out research in which the similarity of the criteria that students adopt when faced with mathematical concepts is clear. Occasionally, students employ so-called “prototype examples” of the function concept in their mind, such as: a function is like$\mathbf{y = x_{two}}$, or a polynomial, or 1/x, or a sine function. Bakar and Tall (1991) also show us that:

> When asked whether a graph is a function, in the absence of a function definition, the mind attempts to respond by resonating with these mental prototypes. If there is resonance, individual experiences lead to a positive response. If there is no resonance, the experiences lead to confusion, searching the mind for meaning to the issue, trying to formulate the reason for the failure to obtain a mental match. (BAKAR and TALL, 1991, p. 02).

The complexity of the concept of function is an idea that spans centuries. For Bakar and Tall (1991), the student cannot build the abstract concept of function without having examples of the function in action; It is necessary to create an approach that makes the models developed by students as appropriate as possible. Tall (2000-a) shows the “Function Machine” as a viable choice:

> A likely good candidate is the machine function as an inbox and outbox. This incorporates icons, visual aspects, presenting both an object status and the input and output process. The usual function representations (table, graph, formula, procedure, verbal formulation, etc.) can also be seen as modes of representation or internal calculation of the input and output relationship (TALL, 2000-a, p. 03).

According to Tall (2000-a) the function machine is a resource of great importance in understanding mathematical concepts, however it is normally used as the “what is my rule” problem, so that students can find the mathematical sentence that expresses the function rule. In this way, an epistemological obstacle arises in that all functions are stated by a formula. The function machine provides students with a mental image of a machine that can be used to explain and name various processes without the need to have an explicit process defined.

According to Tall (2000-a), the function machine in this broader situation is a version that involves the most complete concept of function. It can be thought of and represented in various ways that directly link human understanding, enabling understanding of profound ideas in a simple way. We consider that the function machine is a relevant tool in the process of learning the concept of function, as it significantly promotes learning and develops our students' reasoning. In this way, we aim make our contribution to something important and stimulating, so that learning has meaning in the daily lives of our students.

5 CHOOSE MATHEMATICAL GAMES

Fiorentini and Miorim (1990) consider that the difficulties presented by teachers and students in the process of teaching and learning Mathematics are known to everyone. On the one hand, the student has difficulty understanding the mathematics that the school teaches him, on the other hand, the teacher, aware that he is unable to achieve the desired results with his students, seeks new methods that can improve this situation.

Currently, with competition from the digital world full of electronic resources and social networks that students have access to daily inside and outside the school environment, it is becoming increasingly difficult to motivate students to be interested in what is being studied. Therefore, the best way to overcome these difficulties is to use play.

In this sense, Macedo (2000) advises us that the use of games provides a significant experience for children, both in terms of school content and the development of skills and abilities.

We agree with Kishimoto (1997) when he states that games and activities stimulate the following areas of child development: sensory perception, visual perception, auditory perception, body schema, time-space structuring, memory, attention, imagination, creativity, language and sociability. Therefore, we can ensure that the game develops the mental, affective, physical, reaching the social, in the recognition of other aspects. Playfulness leads to a real understanding of potential, limitations, capabilities and conflicts, but for this to occur Cunha (2012) reiterates that it is necessary:

> [...] a) motivate students for activity; b) encourage student action; c) propose activities before and after the game; d) clearly explain the rules of the game; e) encourage cooperative work between colleagues in the case of group games; f) try not to correct errors directly, but to propose questions that can lead students to discover the solution; g) encourage students to create their own schemes; h) encourage students to make decisions during games; i) encourage students' mental activity through proposals that question the concepts presented in the games; [...] (CUNHA, 2012, p.97).

For Kishimoto (1997), the initial actions of teachers supported by constructivist theories were to transform teaching environments rich in quantity and variety of games, so that students could discover concepts related to game structures through their use.

Almeida (1984) tells us about the relevance of games in education in relation to didactic advantages, stating that such resources have become essential for promoting meaningful learning, as a result of which several theories considered difficult when put through games have proven to be easier to understand.

Lara (2003) highlights that play is an important activity to instigate social life and constructive activity. It is an opportunity to live fully in a climate of participatory learning, in an interactive and creative way in which students and teachers seek to build a productive bond, through which everyone is strengthened in an environment of reciprocal discoveries, in order to achieve objectives proposed and desired.

According to Kishimoto (1997), through games the progress of learning and development takes place, so it can be considered an important ally in school teaching practices, as placing the student in game situations can be a good method. to bring you closer to the cultural content that will be presented at school, in addition to stimulating the development of new cognitive structures.

Confirming this idea, in the High School PCNs (BRASIL,1999, v..3), we see that in addition to being a sociocultural object in which Mathematics is present, the game is a common activity in the development of basic psychological processes. Through it, students not only experience situations that repeat themselves, but they also learn to deal with symbols and think by analogy. When they see these similarities, they become language producers, they start to create conventions, they are able to submit to rules and give explanations.

In line with Huizinga (1999), we state that with the use of games it is possible to develop in students, in addition to mathematical skills, their concentration, curiosity, group awareness, companionship, self-confidence and self-esteem. Thus, in the classroom, the game becomes a preparation for the young person for the tasks that life will require later, it is an indispensable balance exercise for the individual.

According to Huizinga (1999), the game is important because it stimulates the construction of reasoning schemes through its activation, as it encourages the search for solutions through voluntary effort. The use of games in Mathematics classes not only allows the student to build qualitative or logical relationships – learning to reason and ask questions – but also socialization. It becomes a great pedagogical resource in which problematization present provides challenges to the student, encouraging them to seek different processes that lead them to new strategies to find the solution to the calculations present in the activity.

For Lara (2003), if we idealize Mathematics teaching as a process of repetition, training and memorization, we will create a game as just another type of exercise. However, if we design this teaching to be a moment of discovery, creation and experimentation, the game will be seen not only as an instrument of recreation, but essentially as a transport for the construction of knowledge.

According to Rizzo (1996), when observing a game, there is no prior knowledge of the player's action. Uncertainty will always be present. Personal motivations and internal factors determine the player's action, as well as external stimuli, such as the conduct of other partners. When playing and discussing a game, several concepts are reconsidered, as well as various aspects of knowledge are increased and deepened. According to Grando (1995):

> The game is characterized as one that incorporates the mathematical structure, providing a concrete and manipulative representation to support and demonstrate what is behind Mathematics. Thus, aspects related to the game's pedagogical action provide a mathematical discussion that aims, above all, at the student's development and their understanding and relationship with the reality that surrounds them. If the child feels in doubt for any logical or linguistic reason about the mathematical concept, they can resort to the concrete (game) to check and support what they are thinking (GRAND, 1995, p. 105).

For Grossi (2000), it is necessary to learn to lose in educational games, since, to truly learn, it is mandatory to renounce hypotheses that were considered authentic in order to move on to others that are more consistent and more complete.

According to Macedo (2000), any game can be used when the objective is to present activities that benefit the acquisition of knowledge. The problem is not in the material used, but in the way it is exploited. This indicates that the action of playing must be involved and guided, in order to relate a set of intentional actions integrated into the system as a whole.

6 THE ROLE OF THE TEACHER IN THE USE OF MATHEMATICAL GAMES

The teacher must constantly assess the level of interest that the student will have in each game, in order to determine whether or not it will lead to the development of reasoning and cooperation. Knowing the type of challenge that the game presents to the student, it is essential that

The teacher seeks to combine theory and practice so that there is increasingly deeper and more balanced work, with games relevant to the development of students.

The evaluation and choice of the most appropriate games for each group must be carried out with discretion and, whenever possible, include the analysis of the student's participation, and those that do not have significant content that mobilizes the student's thought processes must be excluded.

According to Kamii (1991), a good game is not one that the participant can necessarily master “perfectly”. The important thing is that the student can play in a logical and instructive way for himself and his group.

An important rule when applying games in the classroom requires the possibility for students to evaluate the results of their actions for themselves. It is necessary to avoid any situation of indecision, so that, faced with a failed result, the student can determine where he went wrong and exercise his intelligence in solving problems and thus build relationships between different types of action and different types of reaction of an object.

The active participation of players in a game will depend on everyone's ability to get involved, which will depend on their level of development. This is active participation regarding the mental activity that will be performed by students that involves cooperation between players, whether observing, acting or thinking. According to Kamii (1991), the suggestion of group games is not advocated simply so that children learn to play certain games, the important thing is that the game provides a context that stimulates the student's mental activity and their ability to collaborate, be it player or not, according to the previously established rules.

Lara (2003) classifies mathematical games into construction games, training games, in-depth games and strategic games. Construction games are those that bring to the student a subject that they are unfamiliar with, making them feel, through the manipulation of materials or questions and answers, that they will need a new tool, or if we prefer, new knowledge to solve. given situationproblem proposed in the game. According to Lara (2003), training games can help develop faster deductive or logical thinking. Through repetitive exercises so that the student realizes that there is another path to resolution that could be followed, making their possibilities for action and intervention greater.

According to Lara (2003), another type of game is the so-called in-depth games. After the student has constructed or worked on certain content, the teacher must propose situations in which the student can apply what they have learned. Problem solving is a very appropriate type of activity for this deepening, and such problems can be presented in the form of games. The last type of game are the so-called strategic games that lead the student to create action strategies to better perform as a player, so that he has to raise hypotheses and develop systemic thinking, being able to think of several alternatives for solving problems. a certain problem.

The bibliographic study justifies the choice of educational games in teaching the concept of function. In summary, the game in the classroom is highly recommended, because according to Macedo (2000), the game enables and facilitates the learning process, as the subject is led to think, reflect, make predictions and interact. -relate objects and situations.

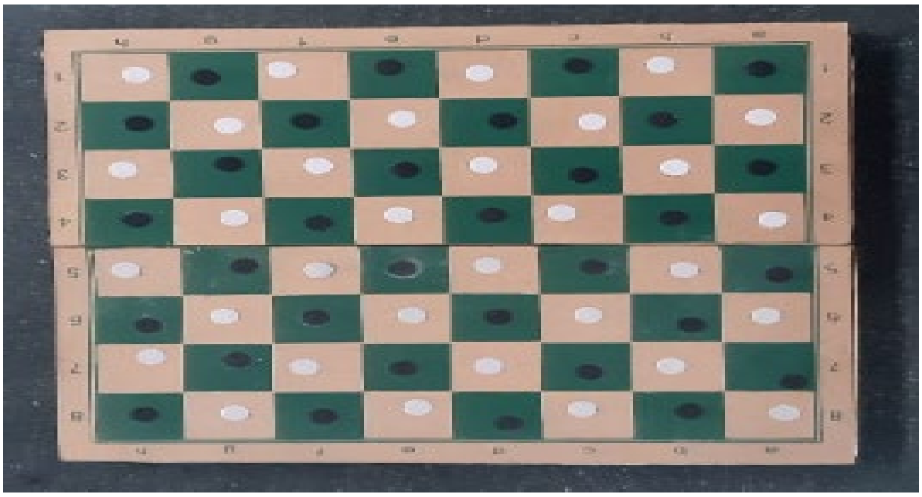
7 GAMES THAT WILL BE USED

For the development of this research, games were built and adapted based on the theories of David Tall and Shlomo Vinner, who in their studies used the idea of the Function Machine to introduce the concept of function. They thought of the function machine as an input and output machine, where an input was placed, and based on a stipulated function, an output was determined. Considering this idea of a machine, we tried to use different types of games that would provide students with different ways of working with the concept of function, both through symbolism and numerical forms.

To carry out this research involving mathematical games as a facilitating tool in teaching function, we will describe the games and their rules. When making the games, material was used so that they were very colorful so that students would feel attracted to participate in the activities. The content presented in each game is different, but they all work with the concepts involved in the function content. The games made are called: Adapted Checkers Game (Cartesian Coordinates), Figures Function Machine (Regional Elements), Find the Exit Function Machine, Determine the Function Function Machine, Naval Battle with Cartesian coordinates, Function Concept Track, Functional Stop, Amazonas Gives Luck and Winding Path.

Adapted Checkers Game (Cartesian Coordinates)

Figure 1- Adapted Checkers Board (Cartesian Coordinates)



Source: Researcher's personal archive

In this adaptation of the Game of Checkers, we define the rules of the game: each square will be identified by an ordered pair of letters and numbers. The letters indicate the columns and the numbers represent the lines, the pieces will be placed on opposite sides, occupying the dark spaces on the board.

The students, in pairs, will start the game and make the moves and must always note the “start box” and the “arrival box”. The winner will be the player who “captures” all of the opponent's pieces and has correctly written all the points found.

Function Machine Figures Game (Regional Elements)

Figure 2. Function Machine game board (Regional Elements)

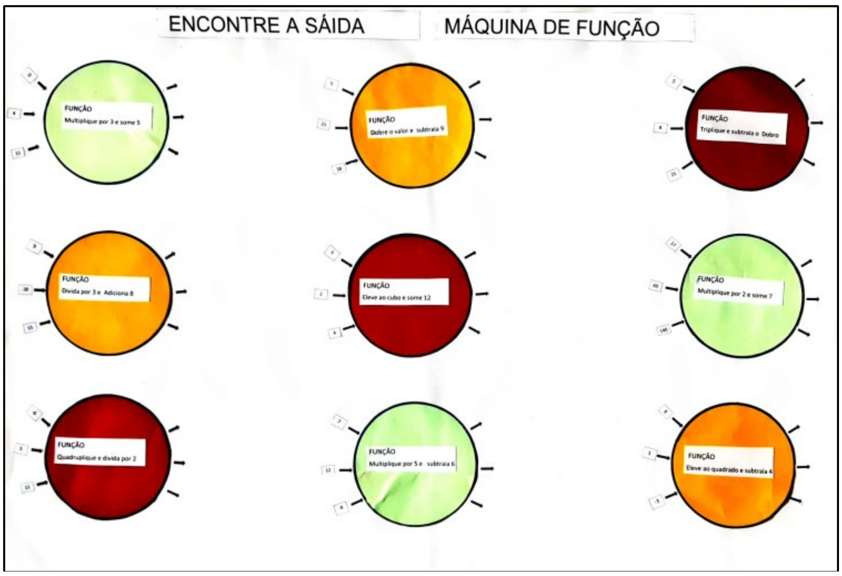
Source: Researcher's personal archive

According to the definition presented by Lara and according to David Tall and Shlomo Vinner. With this game, we intend to create a conceptual image about the concept of function and by using regional figures familiar to students, we seek to develop the concept of Function through symbolic representations, thus establishing a conceptual image.

The Function Machine (Regional Elements) game presents four distinct situations, in which an input, a function and an output are present. In each function there are changes in the figures relating to the outputs.

Game Find the Exit Function Machine

Figure 3. Board for the Find the Exit Function Machine Game.

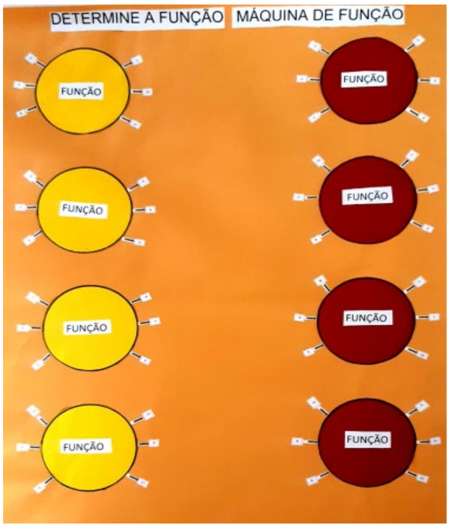


Source: Researcher's personal archive

The game presents different situations: each one presents an input, which contains numbers, and the function is indicated. The question will be asked what the solution will be for each situation indicated.

Game Determine the Function Function Machine

Figure 4. Game Board Determine the Function Function Machine

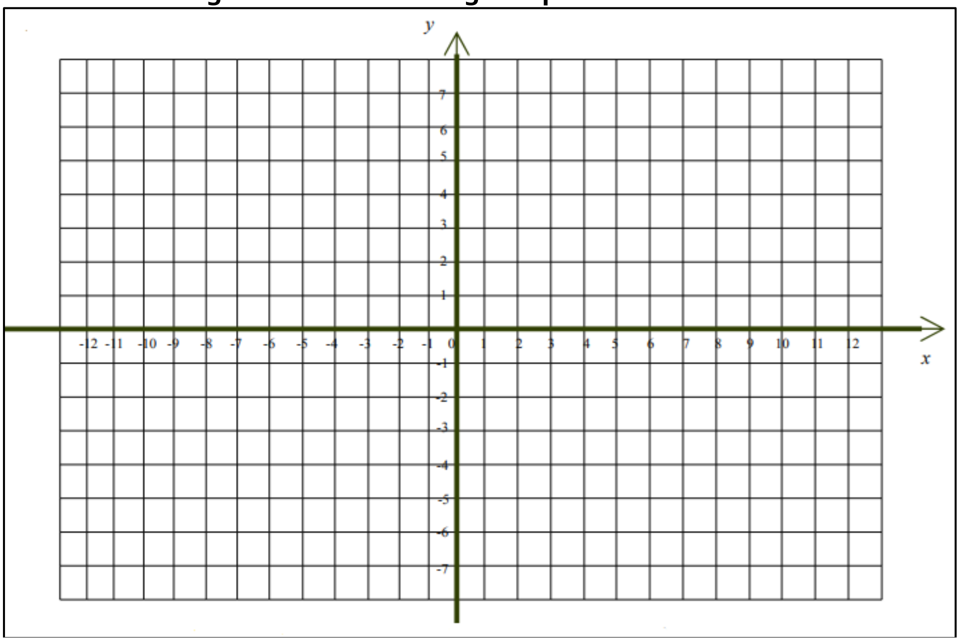


Source: Researcher's personal archive

We established the objectives for this game: to develop the concept of Function through of numerical representations and discover the functions presented in each situation.

Naval Battle Game with Cartesian Coordinates

Figure 5. Naval Battle game plan model

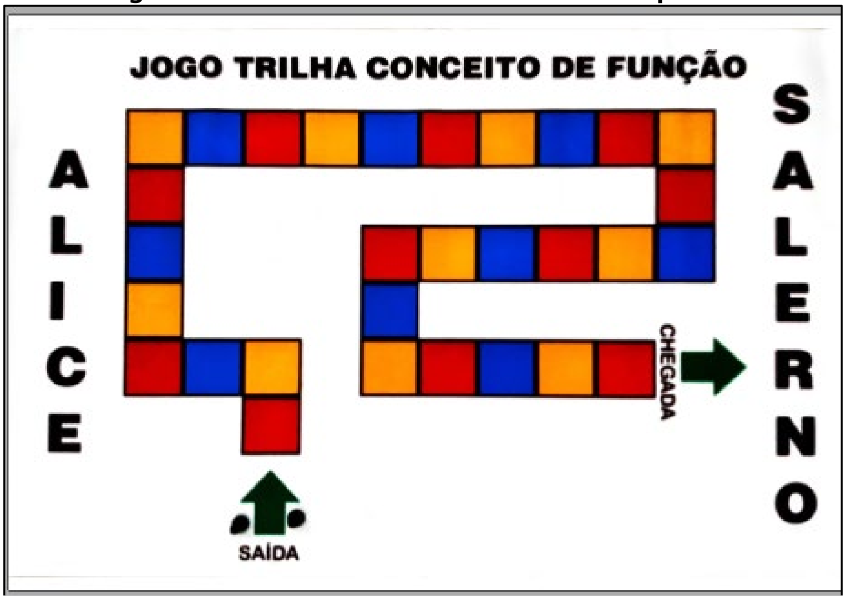


Source: Researcher's personal archive

This game was designed to make the student familiar with the Cartesian plane for use in the concept of function.

Game Trail Function Concept

Figure 6. Game Board Track Function Concept



Source: Researcher's personal archive

This game is classified as a 'deepening game', as it will be applied after the teacher has worked with the students on the concept of function. In the application, the student has the opportunity to solve problem situations with three levels of difficulty, aiming to contribute to a deeper learning. This game aims to: a) recognize the different ways of representing a function, written; numeric; b) recognize the law that relates the variables; c) read and interpret tables and graphs and d) determine the zero of a function.

Functional Stop Game

Figure 7. Functional Stop Game Board

Rodada	Coeficiente Angular	Coeficiente Linear	Crescente / Decrescente	Função	F (2)	F (0)	F (-1)	F (-2)	F(x)=0	Total
1ª										
2ª										
3ª										
4ª										
5ª										
6ª										
7ª										

Source: Researcher's personal archive

This game is classified as an 'in-depth game', as it will be applied after the teacher has worked with the students on the concept of function and some of its elements. When applying this game, the student has the opportunity to deepen their knowledge about functions, aiming to contribute to the retention of learning.

Based on the structure of the classic Stop game, players, already with the necessary information about functions, will fill in the information about them regarding some numbers drawn among the participants in each round.

Amazonas Game Gives Luck Functions

Figure 8. Amazonas Gives Luck Game Board

Source: Researcher's personal archive

Held in pairs, the Amazonas Gives Luck Game aims to encourage students to solve problems related to functions. It is represented by a track where each team has its departure and arrival. Each player rolls the dice and moves the corresponding number of spaces. Every time they stop at the house with the question mark, the player picks up a question and answers it; If it's correct, you can move two spaces forward and if you make a mistake, you stay in the same place. When you stop at the space with the lucky Amazonas flag, you can advance three spaces. Whoever finishes the trail first will win the game.

Winding Path Game Function

Figure 9. Winding Path Game Board

Source: Researcher's personal archive

Caminho Sinuoso is a game composed of a trail that is slightly different from traditional trails and its objectives are to verify whether the graphs represent functions and to recognize the domain and image of the functions and analyze the increasing and decreasing functions represented by graphs. The game will feature three colors of cards: Blue, yellow and red, which will present questions involving different topics about functions. On the blue card, if you ask the question correctly you can move two spaces forward and if you make a mistake you have to go back three spaces. On the yellow card, if you get the question right you will move forward three spaces and if you get it wrong you will move back two spaces. On the red card, if you get it right you will move forward four spaces and if you get it wrong you will move back two spaces.

Based on the studies and research carried out, the constructions of various types of games were made, which consist of ideas based on the concept of function machine, in which the notion of input and output is used so that the different concepts involving functions. In creating the games, both symbolic representation and numerical representation were used. In some games we aimed for students, through group debate, to determine the function that was being represented in a given situation, while in other games the students had to determine the way out of the situation presented, thus developing the construction of conceptual images. , because according to theorists Tall and Vinner (1981) during these mental processes of remembering and manipulating a concept, several associated processes are brought into play, consciously and unconsciously thus affecting meaning and use.

With the application of the games we verified that there was an evolution in the conceptual image in relation to the content of function on the part of the students, the conceptions that the students had about the concept of function were modified with this differentiated work, aiming to constitute meaningful learning through the use of mathematical games. The games were developed to work on the concepts involved in the 1st and 2nd grade Function content. The results obtained through carrying out this research prove that, through practical activities, and especially the use of mathematical games, consistent results can be achieved in relation to the acquisition of knowledge by students, as this way they feel motivated acquire new learning every day.

FINAL CONSIDERATIONS

Considering the theoretical framework, the research problem of this work was proposed: games as facilitating tools in teaching functions in a public school in Manaus.

Based on the authors Tall, Vinner, Lara and Grando, among others, we conclude that games can and should be used as one of the mediating resources in the Mathematics teaching and learning process. Its use configures the possibility of replacing the traditional methodology that only uses brushes, blackboards and textbooks as instruments, with a methodology that makes learning mathematical content interesting, including functions, an active methodology in which we can motivate student learning in a playful way and make the entire process much more dynamic and attractive. To try to solve the learning problem of students in a 1st grade class at Escola Estadual Prof.ª Alice Salerno Gomes de Lima, we initially used the function machine to introduce a visual representation for the concept of function, making learning more consistent, as it operates as an input and output process, where for each element of the input there is a single output to be related. To this end, we adapted a series of games for teaching mathematics, which lead students to reflect on the different concepts surrounding the content of Function.

We know that the difficulties encountered by teachers and students in the process of teaching and learning Mathematics are great. Therefore, a good way to overcome these obstacles is through the use of playful material. However, the teacher must strive to constantly assess the student's level of interest in each game, staying alert to check whether or not the game will lead to progress in reasoning and cooperation.

We verified through carrying out the research that, in relation to the Function content, there are several aspects worked on, such as the graph and the formula, but not all of them are understood. The essence of Function as well as some of the ideas involved are often overlooked. Students are not able to pay attention to the relationship that exists between the graph and the implicit functional process. This is due to the failure to reconcile the mental prototypes that students have with the graphical representations of function. Consequently, students understand the function graph as a static object having no relation to learning.

Analyzing the results achieved in the pre-test and post-test, we verified that many aspects relevant to our research obtained satisfactory results. The conceptual evolution in relation to the Function content on the part of the students was verified.

Using mathematical games, students from a 1st grade class at Escola Estadual Prof.ª. Alice Salerno Gomes de Lima managed to establish a conceptual image, visualizing a function as an input and output machine.

We noticed that through the activities that were carried out there was an evolution in the conceptual images that the students at Estadual Prof.ª. Alice Salerno Gomes de Lima had graphs, expressions and diagrams in relation to the content of the function, which were discussed and analyzed, causing images such as that "function is only represented by graphs and straight lines" to be modified.

REFERENCEs

AKKOÇ, H.; TALL, D.**A Mismatch between Curriculum Design and Student Learning:** The Case of the Function Concept. To be presented at the British Colloquium of Mathematics Education, Warwick. 2005.

ALMEIDA, PN**Playful Dynamics: Pedagogical Games**. São Paulo: Loyola, 1984.

ANDRETTA, M.; GRASSESCHI, M.; SILVA, A. PROMAT:**Math workshop project** – 8th grade. São Paulo: FTD, 2002.

ALVES, Eva Maria Siqueira.**Playfulness and teaching mathematics: a possible practice**. Campinas, SP: Papirus, 2001.

BAKAR, MN; TALL, D.**Students' Mental Prototypes for Functions and Graphs**. Int. J. Math Ed Sci&Techn, 1991.

BARROS. M. &PALHARES. P.**Emergence of Mathematics in Kindergarten**. Porto: Porto Editora Ltda. 1997.

BORIN, J.**Games and Problem Solving:**a strategy for mathematics classes. São Paulo: IME – USP. 1996

BRAZIL. Ministry of Education. Secretariat of Fundamental Education.**National Curriculum Parameters: Mathematics**. Brasília, 1997.

BRAZIL. Secretariat of Fundamental Education.**National Curricular Parameters: Mathematics.**Brasília: MEC/SEF, 1997.

BRAZIL. Ministry of Education. Secretariat of Secondary and Technological Education.**National Curricular Parameters:**High school. Brasília, 1999.

BRAZIL, Secretariat of Secondary and Technological Education.**PCNEM:**National Curricular Parameters for Secondary Education. Brasília: MEC, 2002.

BRAZIL, Secretariat of Basic Education.**Curricular Guidelines for High School: Natural Sciences, Mathematics and their Technologies**. Brasília, MEC, 2006.

CARVALHO, PCP A “domestic” problem.**Mathematics Teacher Magazine** (RPM), Rio de Janeiro, n. 32, SBM, p.1-9, 1996.

DEMO, Pedro.**Educate through research**. São Paulo: Associated Authors, 2002.

FRANCE, Â. AIR**Playing and learning Mathematics with concrete materials, in the 1st year of high school at Colégio Estadual Carlos Gomes.**2012. 60 f. (Didactic-pedagogical production) – Educational Development Program of the State Department of Education of Paraná, State University of Northern Paraná, Jacarezinho, 2012.

FREIRE, P.**Pedagogy of autonomy**. São Paulo: Paz e Terra, 1996.

FIORENTINI, D.; MIORIM, M. Â.**A reflection on the use of concrete materials and games in Mathematics Teaching**. SBEM Bulletin – SP, Year 4, nº 7, 1990.

FLEMMING, Diva Marilia; COLLAÇO DE MELLO, A. Claudia.**Creativity Didactic Games**. São José: Saint-Germain, 2003.

FLEMMING, DM, LUZ, EF, Mello, ACC**Trends in mathematics education**-2nd ed. - Palhoça: Unisul Virtual, 2005.

GOLBERT, CS;**New directions in learning mathematics**. Porto Alegre: Mediation, 119. 2002.

GOLDENBERG, M.**The art of research:**how to do qualitative research in Social Sciences. 8th ed. – Rio de Janeiro: Record, 2004.

GRANDO, R.C.**Mathematical knowledge and the use of games in the classroom**. Doctoral thesis. Campinas, sp. Faculty of Education, UNICAMP, 2000.

GRANDO, R. Celia.**The game [and] its methodological possibilities in the mathematics teaching-learning process**. 1995. 175f. Dissertation (master's degree) - State University of Campinas, Faculty of Education, Campinas, SP. Available at: http:// www.repositorio.unicamp.br/handle/REPOSIP/253786>. Accessed on: 19 Jul. 2018.

GROSSI, EP;**New way to teach Mathematics:**starting with the division. Brasília: Documentation and Information Center, 2000.

HUIZINGA, J.**Homo Ludens:**the game as an element of culture – 1938. Translated by JP Monteiro. São Paulo, Perspectiva, 1971.

HUIZINGA, J.**Homo, Ludens**. Editora Perspectiva SA São Paulo: 1999.

IEZZI, G.; DOLCE, O.; DEGENSZAJN, D. and PÉRIGO, R.**Mathematics: single volume – 2nd grade**. São Paulo: Current, 1997.

IEZZI, G.; MURAKAMI, C.;**Fundamentals of Elementary Mathematics**–8th edn - São Paulo: Ed. Saraiva, 2004.

KAMII, C.; DEVRIES, R.**Group games in early childhood education: Implications of Piaget's Theory.** Translated by Marina Célia Dias Carrasqueira. São Paulo: Cultural Trajectory, 1991.

KISHIMOTO, TM (org.).**Game, toy, play and Education**. 2nd ed., SP: Cortez, 1997.

LARA, IC Machado de.**Playing with Mathematics**. - 1st edition – São Paulo: Rêspel, 2003.

LOPES, Maria da Glória.**Games in Education:**create, make, play. São Paulo: Cortez, 3rd ed. 2000.

MACEDO, L.; MACEDO, ALSP; STEPS, NC**Learning from games and problem situations.** Porto Alegre: Artes Médicas Sul, 2000.

MARKOVITS, Z.; EYLON, B S.; BRUCKHEIMER, M.**Students' difficulties with the concept of function**. IN: COXFORD, AF; SHULTE, AP The ideas of algebra, São Paulo: Atual, p. 49-69, 1995.

MENNA, BM**Mathematics and Sex Education:**modeling the phenomenon of absorption/elimination of daily oral contraceptives. Masters dissertation. PPG-Mathematics Teaching, UFRGS, Porto Alegre. 2008.

MOREIRA, MA**Learning theories**. São Paulo: EPU. SEDUC-RS. 1999.

NOGUEIRA, Cléia Maria I. Trends in school Mathematics Education: student-teacher relationships and mathematical knowledge. In: ANDRADE, Doherty; NOGUEIRA, Cléia Maria Ignatius. org. **Mathematics Education and fundamental operations.**Maringá: EDUEM, 2005.

OLIVEIRA, Sandra Alves de.**Playfulness as motivation in Mathematics classes**. Pedagogue and specialist in Mathematics and Statistics, professor at the Department of Education of Guanambi, BA, Uneb. Email address: soliveira4@hotmail.com Article published in issue no. 377, newspaper Mundo Jovem, June 2007, p. 5.

PONTE, JP**The concept of function in the Mathematics curriculum**. Education and Mathematics Magazine, APM, Portugal, n.15, p. 3-9, 1990.

RIZZO, G.**Smart Games.**The Construction of reasoning in the Natural School. RJ.: Bertrand Brasil, 1996.

SIERPINSKA, A.**On understanding the notion of function**. In: Dubinsky, E.; Harel, G. (Ed.). The concept of function: Elements of Pedagogy and Epistemology (Notes and Reports Series of the Mathematical Association of America, vol. 25). P. 25-58. 1992.

SMOLE, KS; DINIZ, MI; PESSOA, N.; ISHIHARA, C.**Mathematics Notebooks: Mathematics games for 1st to 3rd year**. Porto Alegre: Artmed, 2008.

TALL, D.; VINNER, S. Concept image and concept definition in mathematics, with special reference to limits and continuity, Educational Studies in Mathematics, Dordrecht, vol. 3, no. 12, p. 151-169, 1981.

TALL, D. The Transition to Advanced Mathematical Thinking.**Grouws DA**(Ed). Handbook of Research on Mathematics Teaching and Learning, ed. New York: Macmillan, p. 495-511. 1992.

TALL, D. The Transition to Advanced Mathematical Thinking: Functions, Limits, Infinity, and Proof. **Handbook of Research on Mathematics Teaching and Learning.**New York: NCTM, 1992.

TALL, D.; McGOWEN, M.; MAROIS, P. The Function Machine as a Cognitive Root for building a rich conceptual image of the Function Concept.**Proceedings of PME-NA**, 2000-a.

TALL, D.; McGOWEN, M.; MAROIS, P.**Using the Function Machine as a Cognitive Root.** Proceedings of PME-NA, 2000-b.

TALL, D.; DeMAROIS, P. Facets and Layers of the Function Concept.**Proceedings of PME** 20, Valencia, vol. 2, 297–304. 1996.

TALL, D.; VINNER, S. Concept image and concept definition in mathematics, with special reference to limits and continuity,**Educational Studies in Mathematics,**Dordrecht, vol. 3, no. 12, p. 151-169, 1981.

TALL, D. The Transition to Advanced Mathematical Thinking.**Grouws DA**(Ed). Handbook of Research on Mathematics Teaching and Learning, ed. New York: Macmillan, p. 495-511. 1992.

VINNER, S. Conflicts between definitions and intuitions: the case of the tangent. In: CONFERENCE OF THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION, 6, 1983, Antwerp.**Proceedings**...Antuepia: AntwerpUniversity, p. 24-28, 1983.