



## Teaching Learning aimed at solving mathematical problems

*Learning instruction oriented towards solving mathematical problems*

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### Summary

This article aims to investigate, through problem solving, how mathematics education is taught and learned in everyday life. Observing the current scenario, there has been much discussion about the importance of finding ways to motivate students in this subject. It also focuses on the importance of applying the methodology in teaching Mathematics, addressing the resolution of mathematical problems that spark the student's curiosity, and what strategies and methods they can use when faced with really challenging problem situations. This analysis will be based on the thinking of several mathematical authors and researchers.

**Key words:** Learning; mathematics; Problem solving.

### Summary

This article aims to investigate problem solving, how to learn mathematics education in everyday life. Observing the current scenario, there has been much debate about the importance of seeking ways to motivate students in this discipline. It also focuses on the importance of using methodology in mathematics education, addressing the resolution of mathematical problems that sharpen the student's curiosity, which strategies and methods can be used when faced with really challenging situations and problems. This data analysis will be based on the thinking of several mathematical authors and researchers.

**Keywords:** Learn; Mathematics; Problem solving.

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## **1. Introduction**

According to the PCN - National Curricular Parameters (2001), one of the objectives of Mathematics in primary education is: “resolving problem situations, knowing how to validate strategies and results, developing forms of reasoning and processes, such as deduction, induction, intuition, analogy, estimation, and using mathematical concepts and procedures, as well as available technological instruments.” Given this, we began an argument to find out why there is difficulty in learning mathematics, focused on problem solving.

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The problem-solving methodology aims to take the student from their traditional passive classroom stance to an active stance, thereby seeking to deconstruct the notion that mathematics is something ready and finished. The motivation to solve problems allows for a process of investigation that delineates new mathematical properties. Thus, in the search for a solution to the problem, new situations present themselves, which instigate mathematical curiosity. This teaching-learning process of Mathematics in problem solving does not only privilege individual reasoning, but it instills skills in the construction of knowledge and provokes empathy in students in the classroom.

We are aware that solving mathematical problems is one of the barriers to be faced, many students have difficulty identifying the operation to be used, interpreting and understanding what should be calculated. So we can say that the difficulty in solving mathematical problems is not a difficulty within the mathematics discipline, but rather an interdisciplinary difficulty. There are several factors that lead students to have difficulty interpreting texts or problems, the main one that we can highlight is the lack of reading habits.

On the other hand, teaching for the teacher is not just transmitting, transferring knowledge from one head to another, it is making people think, it is stimulating the student to identify and solve problems, helping them to create habits of thought, to this end they dedicate themselves. and focus their attention on leveraging and strengthening their teaching practice, often seeking to model themselves on mathematical thinkers and scholars.

## 2. Theoretical framework

### 2.1. The challenges encountered in learning

Many challenges are present in the teaching-learning process of solving mathematical problems, however we need to establish a coexistence of respect and exchange of experiences between teacher and student, for this it is necessary that the student understands the application usefulness of the subject in everyday life, understanding that mathematical problems are linked daily in our lives, we live with them, therefore they are linked to the world around us. However, the difficulties encountered by students and teachers in the mathematics teaching-learning process are many and well known. On the one hand, the student cannot understand the mathematics that the school teaches, and is often failed in this subject, or even if approved, they experience difficulty in using the knowledge acquired, in short, they cannot effectively access this knowledge. of fundamental importance.

The teacher, on the other hand, aware that he is unable to achieve satisfactory results with his students and having difficulty, on his own, satisfactorily rethinking his pedagogical practice, often seeks new elements, mere recipes on how to teach certain content that, he believes, can improve this situation.

According to Freire, in his book: Pedagogy of Autonomy (1996, p.96).

A good teacher is the one who can, while speaking, bring the student into the intimacy of the movement of thought. His class is a challenge and not a nursery rhyme. Your students get tired and don't sleep. "They get tired because they follow the comings and goings of their thoughts, they are surprised by their pauses, their doubts, their uncertainties."

But, in the end, the problem is always the same: to interest the student, to provoke him to research, to constantly give him the feeling that he discovers for himself what he is being taught. Therefore, the teacher must not force the conclusion, he must let it form spontaneously in the student's head, it is necessary to let him reason, express his thoughts freely, only in this way can he provoke new mathematical learning. It is not possible, in the pedagogical act, to be with the student, without him being with us.

Students, by sharing their intellectual processes, by learning from their own reasoning and those of others, incorporate new ways of thinking and integrating information. These attitudes highlight the social and human role of Mathematics in school. In this way, the teacher must lead the student to problematize, and never to passively absorb the

ideas and information transmitted. Furthermore, to be a good communicator, the teacher must generate empathy, must try to put themselves in the student's shoes to be able to understand their doubts. The teacher needs to be someone who provokes dialogues, reinforces them and harmonizes solution proposals, based on scientific knowledge.

It is therefore not possible to understand the teaching-learning process without understanding the communication process. Therefore, the teacher must try to eliminate any interference in his messages, and to do so, he must minimize noise in order to obtain good attunement on the part of the students. It is always necessary to bear in mind that certain concepts, made evident to the teacher, are not always clear to the students, and not all students have the same ability to understand a given concept. This fact originates from multiple factors, including the age level and intellectual and social background of the students. If students do not have the ability to understand the proposed work presented, then learning will be null and void.

One of the most important implications of Piaget's theory<sup>3</sup> is that the most efficient learning occurs when the teacher combines the complexity of the subject with the cognitive development of students, bearing in mind that not all students in a class are at the same point of intellectual development.

Teachers should recognize that, for many students, learning mathematics involves great anxiety and fear of failure; rather than dismissing mathematics-related anxiety as baseless, teachers should reassure students that they understand the problem and will work with them in order to overcome this barrier. To be efficient, the teacher must determine the level of development of his students, use a strategy that leads to better and easier learning for them, and help them learn according to their abilities.

According to Gagné<sup>4</sup>, success in a type of learning depends on the prerequisites of that knowledge, which are simpler types of learning. Therefore, to solve certain problems, the student must learn specific associations or facts and differentiate them, then they must learn concepts that begin to be general until they become specific. Only then does the student gain knowledge of certain principles that will allow him to solve problems

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<sup>3</sup>PIAGET, Jean. **Six psychology studies**. Rio de Janeiro: Ed. Forense, 1969.

<sup>4</sup>GAGNÉ. A. **how learning takes place**. Rio de Janeiro: To the Technical Book, 1971.

initials. It is, therefore, a very logical process that begins in the general and ends in the particular, starting in the simple and ending in the complex.

## 2.2. Linking mathematics to problem solving

Mathematics is the science of numbers and calculations. Since ancient times, man has used mathematics to make life easier and organize society. Mathematics was used by the Egyptians in the construction of pyramids, dikes, irrigation canals and astronomy studies. The ancient Greeks also developed several mathematical concepts. Currently, this science is present in several areas of society, such as architecture, IT, medicine, physics, chemistry, etc. We can say that in everything we look there is mathematics.

In this way, this knowledge is not a crystallized and immobile science; it is affected by a continuous expansion and revision of its own concepts. Mathematics should not be presented as a closed, monolithic, abstract or disconnected from reality discipline. Over time, it has been linked to different areas of knowledge, answering many questions and needs of man, helping him to intervene in the world around him. In this context, it is appropriate to highlight the thoughts of Bento de Jesus Caraça:

"Mathematics is generally considered a separate science, disconnected from reality, living in the shadows of the office, a closed office where the noises of the outside world do not enter, neither the sun nor the clamors of men. This is only partly true. Without Without a doubt, Mathematics has its own problems, which have no immediate connection with the problems of social life. But there is also no doubt that its foundations, like those of any other branch of Science, are based on reality. come together in the same mother." (CARAÇA, 1975, p. XIV).

That said, we can say that mathematics is, essentially, a creative activity. Formulating and solving problems constitute the fundamental elements of mathematical activity, without solving and formulating problems you cannot do Mathematics, and this is what gives it its creative character. On the other hand, as a result of the internal and autonomous development of Mathematics or raised by needs and demands that are external to it, these problems, and their formulation and resolution, constitute the most important contribution of Mathematics in its relations with the various sciences and other activities. human. Furthermore, at the level of Mathematics teaching, it is considered that problematic situations favor the creation of rich and stimulating learning environments. In particular, problem solving must be seen as fundamental, and not as something that is eventually done at the end of some chapters as an application of mathematical subjects that until

then they were learned. Mathematics is, so to speak, essentially a thought process that involves the formation and application of networks of abstract and logically associated ideas. These ideas often arise from the need to solve problems in science, technology and everyday life.

### **2.3. Mathematical problems according to thinkers and researchers.**

Mathematical Problems: it is the means by which mathematics develops, it is any situation that requires the mathematical way of thinking and mathematical knowledge to solve them.

Several thinkers and researchers have studied or have studied and researched the activity of solving problems. Here we will focus on the contribution Descartes, Dante and Polya brought to learning problem solving.

### **2.4. Descartes' contributions**

The ideas of problem-solving heuristics come from French philosopher and mathematician René du Perron Descartes (1596 - 1650), Descartes' contribution are his ideas about productive thinking that played an important role in his ambitious project of building a general method of problem solving, he sought to explain in detail how, according to his method, it would be possible to solve any problem. In summary, Descartes sees the problem-solving process in three phases:

- 1- Reduce every algebraic problem to a problem containing only equations;
- 2- Reduce every mathematical problem to an algebraic problem;
- 3- Reduce any problem to a mathematical problem.

We can note that Descartes aims to reduce every problem that exists in the world to a mathematical problem, more than that, Descartes' idea was to complete the project of solving problems and still enjoy its benefits.

However, Descartes presents some ideas of value and relevance related to teaching and that can be applied to problem solving, for example: he proposes a heuristic for solving mathematical problems composed of the following steps:

- Draw a figure;
- Clearly identify what you want to find;

- Assign names and values to each of the known measurements and quantities and unknown;
  - Write all relationships between elements in a symbolic way;
  - Apply various techniques to these relationships until you can solve the problem.
- equation.

This heuristic is very specific to all types of mathematical problems and can be used and adapted to solve any problem.

## 2.5. Polya's contributions

According to George Polya's "Problem solving is a practical skill, like swimming, skiing, or playing the piano: you can learn it through imitation and practice. (...) if you want to learn to swim you have to go in the water and if you want to become a good 'problem solver', you have to solve problems".

George Polya (1897–1985) was one of the most important mathematicians of the 20th century. Born in Hungary, he spent most of his time researching at Stanford University in the United States due to the political situation in Europe at the time of World War II. He researched in various branches of mathematics, such as probability and partial differential equations; His greatest contribution, however, is related to heuristics for solving mathematical problems with several publications related to the subject. Polya is one of the mathematicians of our century who considers Mathematics to be an "observational science" in which observation and analogy play a fundamental role; he also affirms the similarity of the creative process in Mathematics and natural sciences.

Polya was the first mathematician to present a heuristic (this word has the same etymological origin as "eureka", the famous Greek word praised by Archimedes when discovering the solution to the problem of stealing gold from the king's crown. Heuristics is the art of discovery, whose term was popularized by the Hungarian mathematician George Polya) of problem solving specific to mathematics. Therefore, Polya represents a reference on the subject, since his ideas represent a great innovation in relation to problem-solving ideas that existed until then. Many of his ideas are reasonable to this day, serving as a foundation for the work of other researchers contemporary to Polya.

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<sup>5</sup>POLYA, J. **The art of solving problems**. Rio de Janeiro: Interciências, 1995.

Looking to organize the problem-solving process a little, Polya<sup>6</sup> divided it into four stages. It is important to highlight that Polya never intended his division to correspond to a sequence of steps to be followed one after the other without it ever being convenient or necessary to go back, or for his division to function as a magic potion to solve mathematical problems.

The four stages of problem solving according to Polya are: 1st stage: Understanding the problem

The first step is to understand the problem. It's important to ask questions. What is the unknown? What are the data? What are the conditions? Is it possible to satisfy the conditions? Are they sufficient or not to determine the unknown? Are there redundant or contradictory conditions? Constructing figures to schematize the situation proposed in the exercise can be very useful, especially by introducing appropriate notation. Whenever possible, try to separate the conditions into parts.

2nd stage: Construction of a resolution strategy

Find connections between the data and the unknown. It may be worth considering ancillary or private problems if a connection cannot be found within a reasonable time. It's important to ask questions. Have you ever encountered this problem or a similar one? Do you know of a similar problem? Do you know any theorems or formulas that could help? Look at the unknown and try to find a familiar problem that has a similar unknown.

3rd stage: Executing the strategy

This is often the easiest step in the problem-solving process. However, most beginners tend to skip this step prematurely and end up failing. Others develop inadequate strategies and end up getting terribly tangled up in execution (and thus end up being forced to go back to the previous step and develop a new strategy).

4th step: Reviewing the solution

You must examine the solution obtained, checking the results and the arguments used. Can you get the solution in some other way? What is the essence of the problem and the method of resolution applied? In particular, can you use the result – or the method – in any other problem? What is the use of this result? Solution review is the step

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<sup>6</sup>POLYA, J. **The art of solving problems**. Rio de Janeiro: Interciências, 1995.



most important according to Polya<sup>5</sup>, as this step provides debugging and abstraction of the solution to the problem.

## 2.6. Dante's Contributions

According to Dante (2000), we can classify problems into several types:

- Recognition Problems: their objective is to make the student remember or identify a concept, a definition, a property, etc.
- Algorithm Problems: These are those that can be solved step by step.
- Standard Problems: The objective is to remember the basic facts through the algorithms of the four fundamental operations are often challenging.
- Application Problems: These are those that portray real everyday situations and that require the use of mathematics to be solved.
- Puzzle Problems: These are problems that challenge most students.

The solution almost always depends on luck or the ease of understanding some tricks, which is the key to the solution. A problem is more valuable to the extent that whoever is proposing to find a solution to the problem is interested in inventing strategies and creating ideas.

According to Dante<sup>7</sup>, in the problem-solving process, some care needs to be taken taken by the teacher, so that the student does not become discouraged when faced with obstacles.

- Show that there are problems that cannot be solved by simply applying algorithms, formulas or other mechanical procedures;
- Show that there are problems that can be solved in several ways

many different;

- Explain that many problems admit more than one answer;
- Explain that there are problems with excess or lack of data;
- Explore the different ways of representing a problem, that is, through drawings, tables and graphs.

From the citations of the thinkers' contributions, we understand that there is a problem when there is an objective to be achieved and we do not know how to achieve that objective. So we can say that there is a problem when there is a result, known or not.

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<sup>7</sup>DANTE, L.R. **Didactics of solving mathematics problems**. São Paulo: Ática, 2000.

All the thinkers mentioned have as a priority the construction of knowledge through doing and thinking.

### **Final considerations**

Problem solving has an extremely important role in teaching at all levels of knowledge, we can use it as a methodological strategy, for this it is worth highlighting the teacher's interest in adopting, being open to carrying out new experiences and discoveries, as a reward for this acquisition of knowledge, we contribute to the student's school life by awakening logical reasoning in them, making them think, dare and create, this makes all the difference in the educator's profile.

Therefore, the methods of the researchers cited in this article seek to contribute to learning, that is, to prepare the student to face new situations, instigate the investigative spirit and make Mathematics classes more interesting and challenging.

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